

NAME _____

SCHOOL _____

2010 Middle School Math Festival**Team Round: 8, 10, 11, and 12**

Unless stated in the problem, answers will be written as an integer or an exact decimal (i.e. do not round). Any problems requiring a common fraction or mixed number for the answer will always require the fractional part to be in lowest terms.

1. Because of the vast distances in space, when astronomers do calculations, they often use special units called Solar Canonical Units, which are defined below.

$$1 \text{ Astronomical Unit (AU)} = 1.496 \times 10^8 \text{ kilometers}$$

$$1 \text{ Solar Time Unit (TU}_{\text{Sun}}) = 5 \times 10^6 \text{ seconds}$$

$$\mu_{\text{Sun}} = 1.327 \times 10^{11} \text{ km}^3/\text{s}^2$$

The velocity of a planet travelling around the Sun is given by the formula:

$$V_{\text{planet}} = \sqrt{\frac{\mu_{\text{Sun}}}{R}}, \text{ where } R = \text{distance from the Sun to the planet.}$$

Since the velocity Jupiter travelling around the Sun is 0.436 AU/TU, how far from Sun is Jupiter. Write your answer in scientific notation, to the nearest tenth.

2. $f(x) = 2x^2 - 3x + 5$ and $g(x) = x - 3$.

Let $A = f(g(-2))$ and $B = g(f(-2))$. Calculate the positive difference between A and B .

3. A right cylinder has base area of 56π and a total surface area of 336π . Calculate the exact volume of the cylinder (in terms of π).

4. Find the sum of four consecutive even numbers if the first number equals the sum of (one-third the second number) plus (the third number minus 76) plus (one half the fourth number).

5. Find the sum of the first five numbers of the following sequences.

$$A(n) = -2 + (n - 2)(-1.6) \quad B(n) = \frac{1}{10}(-3)^{n-1}$$

Calculate the product of the two sums.

6. $A, B =$ the slope and y-intercept of the line containing the points:

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$$(-3.5, 5.2) \text{ and } (4.7, 24)$$

Calculate $A + B$. Write the answer as a mixed number.

7. A plane travels 492 miles in 75 minutes when the wind is behind the plane. The return trip, with the wind blowing towards the plane, takes 112.5 minutes.

Let C = the speed of the plane, with no wind, and D = speed of the wind.

Calculate $\frac{C}{D}$. Write the answer as a decimal.

8. For $x = -\frac{2}{3}$, calculate ABC . Write your answer as a common fraction.

$$A = \left(\frac{64}{125}\right)^x \qquad B = \left(-\frac{8}{27}\right)^{-x} \qquad C = \left(\frac{216}{343}\right)^{x+1}$$

9. Calculate $(Q + R)ST$.

Q and R are the two solutions to $x^2 + 4x + 9 = 0$
 S and T are the two solutions to $12x^2 - x - 6 = 0$

10. A dog is attached to one corner of a 10-foot by 10-foot square shed and is restrained by a 15-foot long leash.

Let C = the area (in ft^2) where the dog is able to roam, in terms of π .

Let D = the perimeter (in feet), including the edge of the shed, where the dog is able to roam, written in terms of π .

Calculate $C + D$, written in terms of π .