

2009 Middle School Math Festival**Team Round: Algebra**

Unless stated in the problem, answers will be written as an integer or an exact decimal (i.e. do not round). Any problems requiring a common fraction or mixed number for the answer will always require the fractional part to be in lowest terms.

1. Because of the vast distances in space, when astronomers do calculations, they often use special units called Solar Canonical Units, which are defined below.

$$1 \text{ Astronomical Unit (AU)} = 4.9 \times 10^{11} \text{ feet}$$

$$1 \text{ Solar Time Unit (TU}_{\text{Sun}}) = 5 \times 10^6 \text{ seconds}$$

Using these units, the Earth travels around the Sun at $1 \text{ AU}/1 \text{ TU}_{\text{Sun}}$. Convert this speed to miles per hour and round your answer to the nearest *hundred*.

2. $f(x) = -13.8x$ and $g(x) = 8x + 3$. Calculate $g\left(f\left(\frac{1}{3}\right)\right) - f\left(g\left(-\frac{1}{4}\right)\right)$.

3. A right cylinder has a diameter of $6\sqrt{2}$ and a volume of $(90\sqrt{2})\pi$. Calculate the total surface area (in terms of π).

4. Find the sum of four consecutive odd numbers if the 4th number is twice the 2nd number minus 21.

5. Find the fifth term of each of the following four sequences. What is the product of the four values? Write the answer as a decimal.

$$4\frac{2}{5}, 3\frac{3}{5}, 2\frac{4}{5}, \dots$$

$$A(n) = -2 + (n - 2)(-1.6)$$

$$89.1, -29.7, 9.9, \dots$$

$$B(n) = \frac{1}{10}(-3)^{n-1}$$

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6. A, B = the slope and y-intercept of the line containing the points $(3, -7)$ and $(-4, 3)$
 C = the slope of a line perpendicular to the line $7x + 2y = 25$

Calculate the sum of $A + B + C$. Write the answer as a mixed number.

7. You have 13 coins, comprised of only nickels and quarters, which total \$1.65. Let N equal the number of nickels and Q equal the number of quarters. A plane travels 300 miles with the wind in three hours but going against the same wind it takes the plane 3 hours and 45 minutes to return to the starting point. Let S equal the speed of the plane (no wind) and W equal the wind speed. Calculate $\frac{SW}{NQ}$. Write the answer as a decimal.

8. For $x = -3$, calculate $ABCD$. Write your answer as a common fraction.

$$A = \left(\frac{2}{3}\right)^x \quad B = -9(-3)^x \quad C = \left(-\frac{1}{4}\right)^x \quad D = 8(3)^x$$

9. Calculate $(Q + R)ST$. Write the answer as a common fraction.

$$Q, R = \text{the two solutions to the equation } 10x^2 + 11x = 6.$$

$$S, T = \text{the two solutions to the equation } 6x^2 = 11x + 10.$$

10. For the following two coordinate points, $(-2.3, 5.8)$ and $(5.2, -4.2)$, calculate the sum of L, M , and N . Write the answer as a decimal.

L = the distance between the two points (to the nearest tenth)

(M, N) = x - and y - coordinates of the midpoint of the two points

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11. You have 260 feet of fencing. Let A = the maximum open area which can be enclosed by the fencing. If you have an open area of 1225 ft^2 which needs to be enclosed, let B = the minimum amount of fencing needed to enclose the area. Ignoring the units, find the sum of $A + B$.

12. A right triangle FGH has sides of length $FH = 10.5$, $GH = 36$, and $FG = 37.5$.

Calculate $\frac{\sin F + \cos F}{\tan F}$. Write the answer as a common fraction in lowest terms.

13. Calculate FGH where $F = \frac{2}{5}\%$ of $\frac{4}{5}$; $3.1 = G\%$ of 20; $70 = 3.5\%$ of H
Write the answer as a decimal.

14. Calculate $A + B + C + D$.

$$A, B \text{ are the two solutions to } |-2k + 3| = 5$$

$$C, D \text{ are the two solutions to } |-2c - 3| - 8 = -3$$

15. A bag of marbles contains 3 red, 4 blue, 5 white, 2 black marbles, and 1 orange marble. Calculate $\frac{P + Q}{R}$. Write the answer as a mixed number with the fraction in lowest terms.

P = probability of randomly selecting one marble and it is **NOT** orange.

Q = probability of randomly selecting one marble and it is red or white.

R = probability of randomly selecting two marbles (at the same time)
and one is black and one is blue.