2009 GEOMETRY TEAM

(1) $P = 260$ feet of fencing. Max area = square so $S = \frac{260}{4} = 65\text{ft}$

$A = s^2 = 4225\text{ft}^2$

Area $= 1225\text{ft}^2$, Min. perimeter = square so $s = \sqrt{1225} = 35\text{ft}$

$B = 4(35) = 140$ $\quad A + B = 4225 + 140 = \boxed{4365}$

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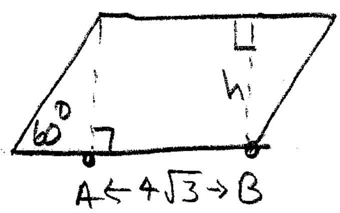
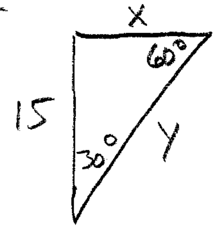
(2)



$h = 15$



$x\sqrt{3} = 15$ so $x = \frac{15}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$x = 5\sqrt{3}$

$y = 2x = 10\sqrt{3}$

Perimeter $= 2(10\sqrt{3}) + 2(4\sqrt{3} + 5\sqrt{3}) = 20\sqrt{3} + 18\sqrt{3} = \boxed{38\sqrt{3}}$

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(3) $(-2.3, 1.8)$ $\quad d_1 = \sqrt{(2.7 - {}^-2.3)^2 + (-3.2 - 1.8)^2} = \sqrt{5^2 + (-5)^2} = \sqrt{50}$

$(2.7, -3.2)$ $\quad = 5\sqrt{2}$

$(-5.3, 0.8)$ $\quad d_2 = \sqrt{(-5.3 - 2.7)^2 + (0.8 - {}^-3.2)^2} = \sqrt{(-8)^2 + (4)^2}$

$= \sqrt{64 + 16} = \sqrt{80} = 4\sqrt{5}$

$d_3 = \sqrt{(-2.3 - {}^-5.3)^2 + (1.8 - 0.8)^2} = \sqrt{3^2 + 1^2} = \sqrt{10}$

Perimeter: $\boxed{5\sqrt{2} + 4\sqrt{5} + \sqrt{10}}$

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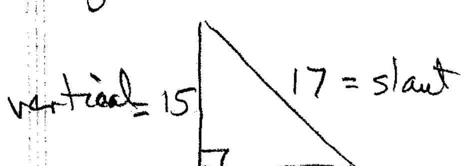
(4) square right perimeter $\quad$ total surface area $= B + 4(\frac{1}{2}bh)$



$B = b^2 = 16^2 = 256$

$4(\frac{1}{2})(16)(15) = 480$

$SA = 256 + 480 = \boxed{736}$

$x = 8 = \frac{1}{2}$ base length

$b = 16 \Leftarrow$

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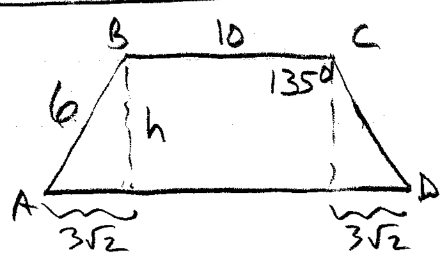
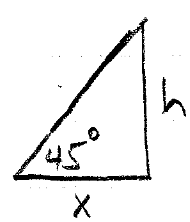
(5)





$h\sqrt{2} = 6$

$h = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 3\sqrt{2}$

$x = h$ so $b_2 = 10 + 6\sqrt{2}$

$A = \frac{1}{2}(b_1 + b_2)(h) = \frac{1}{2}(10 + 10 + 6\sqrt{2})(3\sqrt{2})$

$= \frac{1}{2}(20 + 6\sqrt{2})(3\sqrt{2}) = (10 + 3\sqrt{2})(3\sqrt{2})$

$= \boxed{30\sqrt{2} + 18}$

2009 GEOMETRY TEAM

(6) 100 coins stretch 2 meters = 2000 mm

diameter of 1 coin = 20 mm so $r = 10$ mm

$C = \pi d = 20(3.14) = 62.8$

$A = \pi r^2 = 100(3.14) = 314$

$C + A = \boxed{376.8}$

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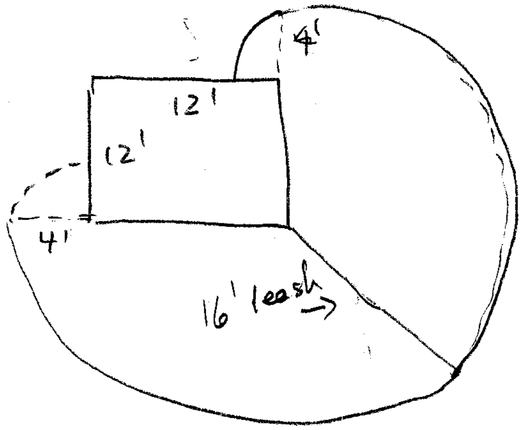
(7)



Large area $= \frac{3}{4}\pi r^2$

$= \frac{3}{4}(\pi)(16)^2$

$= \frac{3}{4}(16)(16)\pi = 192\pi$

2 small areas $= 2\left(\frac{1}{4}\right)\pi r^2$

$= 2\left(\frac{1}{4}\right)(\pi)(4)^2 = 8\pi$

Total area $= 192\pi + 8\pi = \boxed{200\pi \text{ ft}^2}$

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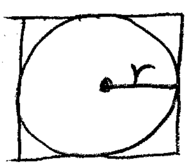
(8)



$s = d$

$A_{sq} = 18$ so $s = d = \sqrt{18} = 3\sqrt{2}$

$r = 1.5\sqrt{2}$

$C = \pi d = 3\sqrt{2}\,\pi$

$A = \pi r^2 = (1.5\sqrt{2})^2\pi = 4.5\pi$

$AC = (3\sqrt{2}\pi)(4.5\pi) = \boxed{(13.5\sqrt{2})\pi^2}$

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(9) $3x - 8y = -2 \Rightarrow$ A = slope = $3/8$

$5x + 4y = 100 \Rightarrow$ B = slope of parallel = $-5/4$

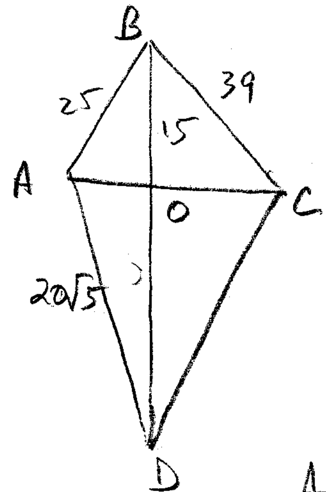
$2y - 5y = -4.3 \Rightarrow$ $m = 2/5$ slope of perpendicular $= C = -\frac{5}{2}$

$A - B + C = \frac{3}{8} - \left(\frac{-5}{4}\right) + \left(\frac{-5}{2}\right)$

$= \frac{3}{8} + \frac{5}{4} - \frac{5}{2} = \frac{3}{8} + \frac{10}{8} - \frac{20}{8} = \boxed{-\frac{7}{8}}$

2009 Geometry TEAM

(10)



$A = \frac{1}{2}d_1 d_2$ $\quad d_1 = AO + OC$

$d_2 = 15 + OD$

$\triangle AOB$ is 3-4-5, so $AO = 20$

$\triangle BOC$ is 5-12-13, so $OC = 24$

for OD: $20^2 + (OD)^2 = (20\sqrt{5})^2$

$400 + (OD)^2 = 2000$

$OD^2 = 1600 \Rightarrow OD = 40$

$A = \frac{1}{2}(44)(65) = \boxed{1430}$

(11) 1 cubit ~ 18 in = 1.5 ft

30 cubit = 45 ft

50 cubit = 75 ft

300 cubit = 450 ft

$V = (45)(75)(450) = \boxed{1{,}518{,}750 \text{ ft}^3}$

(12) $\left.\begin{array}{l}(3,-7)\\(-4,3)\end{array}\right\}$ $m = \frac{3-7}{-4-3} = \frac{10}{-7} = -\frac{10}{7} = A$

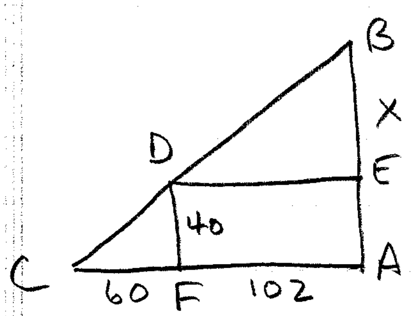
sub. $(-4,3)$: $3 = (-\frac{10}{7})(-4) + b$

$b = 3 - \frac{40}{7} = -\frac{19}{7} = B$

$7x + 2y = 25 \Rightarrow m = -7/2 \Rightarrow$ for $\perp$, $C = 2/7$

$A + B + C = -\frac{10}{7} - (\frac{-19}{7}) + \frac{2}{7} = \frac{11}{7} = \boxed{1\,4/7}$

(13)



$\frac{BE}{DE} = \frac{BA}{CA} \Rightarrow \frac{X}{102} = \frac{X+40}{162}$

$162x = 102x + 4080$

$60x = 4080$

$\boxed{x = 68}$