

2009 ALGEBRA TEST

①  $\frac{4.9 \times 10^{11} \text{ ft}}{5 \times 10^6 \text{ sec}} \times \frac{1 \text{ mile}}{5280 \text{ ft}} \times \frac{3600 \text{ sec}}{\text{hr}} = \boxed{66800 \text{ mph}}$

②  $f(x) = -13.8x$   
 $g(x) = 8x + 3$

$g(f(1/3)) \Rightarrow -13.8(1/3) = -4.6$   
 $8(-4.6) + 3 = -33.8$   
 $f(g(-1/4)) \Rightarrow 8(-1/4) + 3 = 1$   
 $-13.8(1) = -13.8$

$g(f(1/3)) - f(g(-1/4)) = -33.8 - (-13.8) = \boxed{-20}$

③



$d = 6\sqrt{2}$  so  $r = 3\sqrt{2}$

$V = \pi r^2 h = \pi (3\sqrt{2})^2 h = 18\pi h$

$V = (90\sqrt{2})\pi$  so  $h = \frac{90\sqrt{2}}{18} = 5\sqrt{2}$

$SA = 2\pi r^2 + 2\pi r h = 2\pi (3\sqrt{2})^2 + 2\pi (3\sqrt{2})(5\sqrt{2})$   
 $= (6\sqrt{2})\pi + 60\pi = \boxed{(60 + 6\sqrt{2})\pi}$

④ 4 consecutive odd numbers:  $x, x+2, x+4, x+6$

$x+6 = 2(x+2) - 21$

Sum =  $4x + 12$

$x+6 = 2x + 4 - 21$   
 $-x + 17 = -x + 17$

$= 4(23) + 12$

$= 92 + 12 = \boxed{104}$

$23 = x$

⑤ 5th term:  $4\frac{2}{5}, 3\frac{3}{5}, 2\frac{4}{5}, \dots$  subtract  $\frac{4}{5} \Rightarrow 2, \boxed{1\frac{1}{5} = 1.2}$

$89.1, -29.7, 9.9 \rightarrow$  dividing by  $-3 \Rightarrow -3.3, \boxed{1.1}$

$-2 + (n-2)(-1.6) = -2 + (3)(-1.6) = -2 + -4.8 = \boxed{-6.8}$

$\frac{1}{10} (-3)^{n-1} = \frac{1}{10} (-3)^4 = \frac{1}{10} (81) = 8.1$

Product =  $(1.2)(1.1)(-6.8)(8.1) = \boxed{-72.7056}$

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⑥  $(3, -7) (-4, 3) \Rightarrow m = \frac{3 - -7}{-4 - 3} = \frac{10}{-7} = A$

Use  $(3, -7)$ :  $-7 = -\frac{10}{7}(3) + b \Rightarrow -7 = -\frac{30}{7} + b \Rightarrow b = \frac{-19}{7} = B$

$7x + 2y = 25$   
 $y = -\frac{7}{2}x + \frac{25}{2} \Rightarrow m_{\perp} = \frac{2}{7} = C$

$A + B + C = -\frac{10}{7} + \frac{-19}{7} + \frac{2}{7} = \frac{-27}{7} = \boxed{-6 \frac{6}{7}}$

⑤ 13 coins (nickels + quarters) total 1.65  
 $5N + 25Q = 165 \rightarrow N + 5Q = 33$   
 $N + Q = 13 \quad - (N + Q = 13)$   
 $4Q = 20 \rightarrow Q = 5 \text{ so } N = 8$

PLANE:  $d = rt$  } WITH WIND:  $[300 = (S+W)3] \div 3$   
 AGAINST WIND:  $[300 = (S-W)3.75] \div 3.75$   
 $100 = S+W$  }  $2S = 180$   
 $80 = S-W$  }  $S = 90, \text{ so } W = 10$

$\frac{SW}{NQ} = \frac{(90)(10)}{(5)(8)} = \boxed{22.5}$

⑧  $x = -3$   
 $A = \left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$   
 $B = -9(-3)^{-3} = \frac{-9}{-27} = \frac{1}{3}$   
 $C = \left(-\frac{1}{4}\right)^{-3} = (-4)^3 = -64$   
 $D = 8(3)^{-3} = \frac{8}{3^3} = \frac{8}{27}$

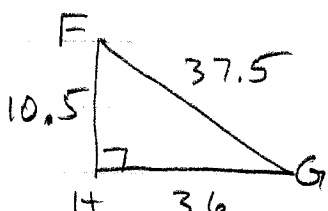
$ABCD = \left(\frac{27}{8}\right)\left(\frac{1}{3}\right)(-64)\left(\frac{8}{27}\right)$   
 $= \boxed{\frac{-64}{3}}$

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9)  $10x^2 + 11x = 6$   $6x^2 = 11x + 10$   
 $10x^2 + 11x - 6 = 0$   $6x^2 - 11x - 10 = 0$   
 $(5x - 2)(2x + 3) = 0$   $(3x + 2)(2x - 5) = 0$   
 $Q, R = \frac{2}{5}, -\frac{3}{2}$   $S, T = -\frac{2}{3}, \frac{5}{2}$   
 $(Q+R)ST = \left(\frac{2}{5} + \frac{-3}{2}\right) \left[\left(-\frac{2}{3}\right)\left(\frac{5}{2}\right)\right] = \left(\frac{4}{10} + \frac{-15}{10}\right) \left[-\frac{5}{3}\right]$   
 $= \left(\frac{-11}{10}\right) \left(-\frac{5}{3}\right) = \boxed{\frac{11}{6}}$

10)  $(-2, 3, 4.8)$  }  $d = \sqrt{(5.2 - -2.3)^2 + (-4.2 - 4.8)^2} = \sqrt{(7.5)^2 + (-10)^2}$   
 $(5, 2, -4.2)$  }  $= \sqrt{56.25 + 100} = \sqrt{156.25} = 12.5 = L$   
 $M = \left(\frac{-2.3 + 5.2}{2}, \frac{4.8 + -4.2}{2}\right) = (1.45, 0.3) = (M, N)$   
 $L + M + N = 12.5 + 1.45 + 0.3 = \boxed{14.25}$

11)  $P = 260$  feet fencing  $\Rightarrow$  max. area = square, so  $S = \frac{260}{4} = 65$   
 $A_{\text{square}} = 65^2 = 4225 \text{ ft}^2$   
 $A = 1225 \text{ ft}^2$   
min. perimeter = square, so  $S = \sqrt{1225} = 35 \Rightarrow P = 4(35) = 140$   
 $A + B = 4225 + 140 = \boxed{4365}$

12)   
 $\sin F = \frac{36}{37.5} = \frac{24}{25}$   $\tan F = \frac{36}{10.5} = \frac{24}{7}$   
 $\cos F = \frac{10.5}{37.5} = \frac{7}{25}$

this is a 7-24-25 triangle (multiplied by 1.5)

$$\frac{\sin F + \cos F}{\tan F} = \frac{\frac{24}{25} + \frac{7}{25}}{\frac{24}{7}} = \frac{31}{25} \cdot \frac{7}{24} = \boxed{\frac{217}{600}}$$

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(13)  $F = \frac{2}{5}\%$  of  $\frac{4}{5} \Rightarrow 0.4\%$  of  $0.8 = (.004)(0.8) = .0032$

$3.1 = 6\%$  of  $20 \Rightarrow 6\% = \frac{3.1}{20} = .155 = 15.5\%$

$70 = 3.5\%$  of  $H \Rightarrow 70 = .035H \Rightarrow H = \frac{70}{0.35} = 2000$

$FGH = (.0032)(15.5)(2000) = \boxed{99.2}$

(14)  $|-2k+3| = 5$

$\swarrow \quad \searrow$   
 $-2k+3=5 \quad -2k+3=-5$

$-2k=2 \quad -2k=-8$

$A=k=-1 \quad B=k=4$

$|-2c-3|-8 = -3$

$|-2c-3| = 5$

$\swarrow \quad \searrow$   
 $-2c-3=5 \quad -2c-3=-5$

$-2c=8 \quad -2c=-2$

$C=c=-4 \quad D=c=1$

$A+B+C+D = -1+4-4+1 = \boxed{0}$

(15)

3 red

4 blue

5 white

2 black

1 orange

15 total

$P = P(\text{not orange}) = \frac{14}{15}$

$Q = P(\text{red or white}) = \frac{8}{15}$

$R = P(2 \text{ marbles; } 1 \text{ black, } 1 \text{ blue}) = \frac{2}{15} \cdot \frac{4}{47} = \frac{4}{105}$

$\frac{P+Q}{R} = \frac{\frac{14}{15} + \frac{8}{15}}{\frac{4}{105}} = \frac{\frac{22}{15} \cdot \frac{7}{42}}{\frac{4}{105}} = \boxed{\frac{77}{2}}$

$= 38.5$