2008 Geometry

① $\left.\begin{matrix}(2,5)\\(4,6)\end{matrix}\right\}$ $m = \frac{6-5}{4-2} = \frac{1}{2}$

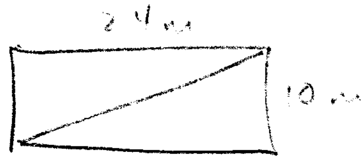
$5 = 2(\frac{1}{2}) + b$
$5 = 1 + b$
$b = 4$

Ⓐ $y = \frac{1}{2}x + 4$

$\boxed{(a)\ (6,7)}$ $7 = \frac{1}{2}(6) + 4 = 3 + 4$ ✓
(b) (3,4) $4 \neq \frac{1}{2}(3) + 4$
(c) (7,6) $6 \neq \frac{1}{2}(7) + 4$
(d) (1,3) $3 \neq \frac{1}{2}(1) + 4$

---

②



Ⓒ

$d = \sqrt{10^2 + 24^2} = \sqrt{100 + 576} = \sqrt{676}$

$\boxed{d = 26\text{ m}}$

Pythagorean Triple ⟹ 5-12-13 ⟶ 10-24-26

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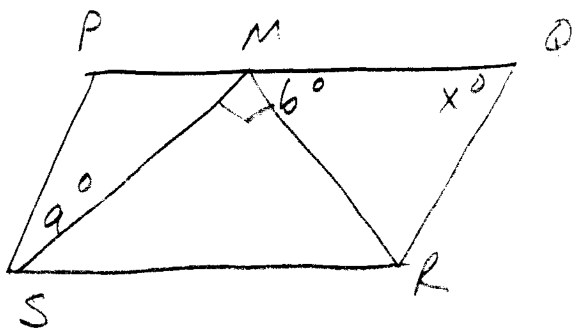
③



Ⓓ

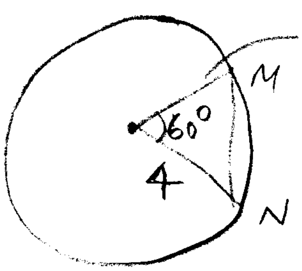
$\angle MRS = b^\circ$ (alt. interior)
$\angle MSR = 90 - b^\circ$
$\angle PSR = a^\circ + 90 - b^\circ = x^\circ$

so; $\boxed{x = 90 + a - b}$

PQRS is parallelogram
$\angle SMR$ = right angle

---

④



equilateral triangle

Ⓒ

$m\,\widehat{MN} = \frac{60}{360}(2\pi r) = \frac{1}{6}(2\pi)(4)$
$= \boxed{\frac{4}{3}\pi}$

2008 Geometry

A

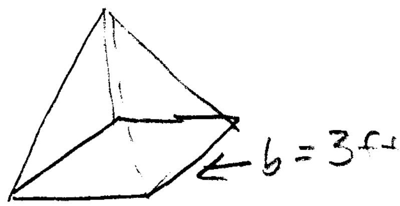
(5) A(-5, -2)
B(-3, 2)
C(-1, -2)

reflected over y-axis => $(x, y) \to (-x, y)$

$$\boxed{\begin{aligned} A' &= (5, -2) \\ B' &= (3, 2) \\ C' &= (1, -2) \end{aligned}}$$

B

(6) Square pyramid



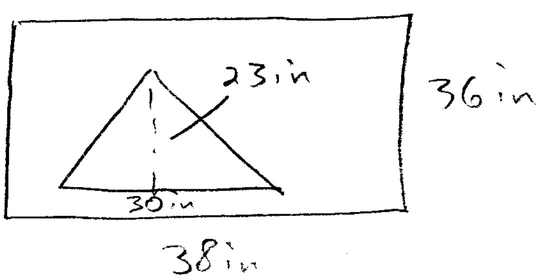
B = Base area = 9 ft^2
Slant height = 8 ft

Total surface area =
$B + 4(\frac{1}{2})(b)(l)$
base length, slant height

$SA = 9 + 4(\frac{1}{2})(3)(8)$
$= 9 + 48 = \boxed{57 \text{ ft}^2}$

C

(5)



$A_{rect} = (38)(36) = 1368 \text{ in}^2$
$A_{tri} = \frac{1}{2}(30)(23) = 345 \text{ in}^2$

$A_{remaining} = 1368 - 345$
$= \boxed{1023 \text{ in}^2}$

D

(7) Trapezoid: A = 160 cm^2
one base = 20 cm
height = 10 cm

$A = \frac{1}{2}h(b_1 + b_2)$

$160 = \frac{1}{2}(10)(20 + b_2)$
$160 = 5(20 + b_2)$
$32 = 20 + b_2$
$\boxed{12 = b_2}$

2008 Geometry

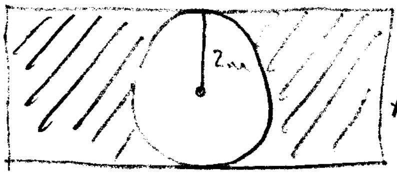
A

9. $C = 52.7$ mm
Find d (nearest tenth)

$C = \pi d$
$52.7 = 3.14(d) \Rightarrow d \cong 16.78$ mm
$\boxed{d = 16.8\text{ mm}}$

C

10.



8 mm

$h = 4$ m

Find A_{shaded}
$r = 2$ mm

$A_{shaded} = A_{rect} - A_{circle}$
$= (8)(4) - \pi(2)^2$
$= 32 - 4(3.14)$
$= 32 - 12.56 = 19.44$
$\boxed{A_{shaded} = 19.4\text{ mm}^2}$

D

11. Regular pentagon. Find measure of interior angle

Method 1: sum of interior angles $= (n-2)180$
$= (5-2)(180) = 540^\circ$

each measure $= 540/5 = \boxed{108^\circ}$

Method 2: sum of exterior angles $= 360^\circ$
each exterior $= 360/5 = 72^\circ$
interior $= 180 - \text{exterior} = 180 - 72 = 108^\circ$

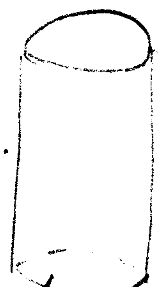
B

12. Box:
Top + bottom $\Rightarrow$ 3" x 9"
Sides $\Rightarrow$ 3" x 12"
Front + back $\Rightarrow$ 9" x 12"

$SA = 2(3)(9) + 2(3)(12) + 2(9)(12)$
$= 54 + 72 + 216$
$\boxed{SA = 342\text{ in}^2}$

D

13.



14 in.

$r = 5$ in

Vase:

$SA = 2\pi r h + \pi r^2$
$= 2(3.14)(5)(14) + (3.14)(5)^2$
$= 439.6 + 78.5$
$= 518.1$
$\boxed{SA = 518\text{ in}^2}$

2008 GEOMETRY

(B) (14) 201 ft long highway
8 ft wide
6 in deep

$V = lwh = (201)(8)(.5)$

6 in = .5 ft

$\boxed{V = 804 \text{ ft}^3}$

(A) (15) ratio of dimensions is 2:1

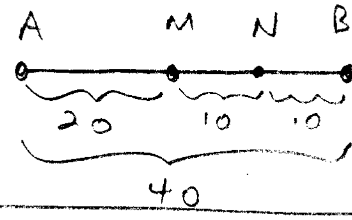
$V_1 = 360 \text{ in}^3$

$V_2 = (1/2)^3(360) = \boxed{45 \text{ in}^3}$

(B) (16) If you live in California, then you need a mountain bike.

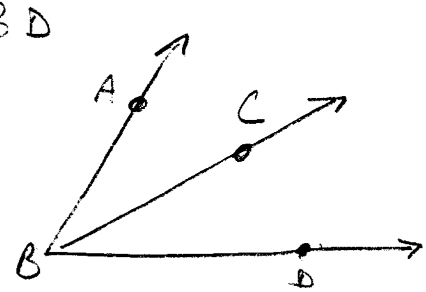
Converse: If you need a mountain bike then you live in California!

(D) (17) M is midpoint of $\overline{AB}$
N is midpoint of $\overline{MB}$
AB = 40, find $\overline{AN}$



$\boxed{AN = 30}$

(C) (18) $\overrightarrow{BC}$ bisects $\angle ABD$
$m\angle ABD = 28y - 6$
$m\angle CBD = 37 - y$
Find y



$m\angle ABD = 2(m\angle CBD)$
$28y - 6 = 2(37 - y)$
$28y - 6 = 74 - 2y$
$30y = 80$
$y = 80/30 = \boxed{8/3}$

(B) (19) A(0,0)
B(4,2)

Find equation of perpendicular bisector.

slope of $\overleftrightarrow{AB} = \frac{2}{4} = 1/2$
slope of $\perp$ bisector $= -2$
midpoint of $\overleftrightarrow{AB} = (2, 1)$

$y = mx + b$
$1 = (-2)(2) + b$
$1 = -4 + b$
$b = 5$

$\boxed{y = -2x + 5}$

2008 Geometry

(20) angles of triangle are $x°$, $5x°$, $(90-x)°$
Find positive difference between largest - smallest.

(C)

$x + 5x + 90 - x = 180$

$5x = 90$

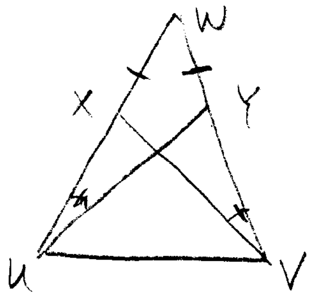
$x = 18$

→ angles = 18, 90, 72

largest - smallest = $90 - 18 = \boxed{72°}$

---

(21)



(D)

What congruence proves $\Delta UYW \cong \Delta VXW$?

$\angle W \cong \angle W$ (reflexive)

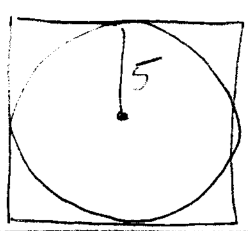
so $\boxed{AAS}$

$\angle WUY \cong \angle WVX$

$WX = WY$

---

(22)



(A)

length of side of the square = 10

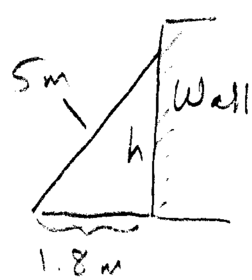
$P_{square} = 4(10) = 40$

$C = 2\pi r = 10\pi \approx 31.4$

$40 - 31.4 = 8.6 \Rightarrow \boxed{\sim 9}$

---

(23)



(B)

$h^2 + 1.8^2 = 5^2$

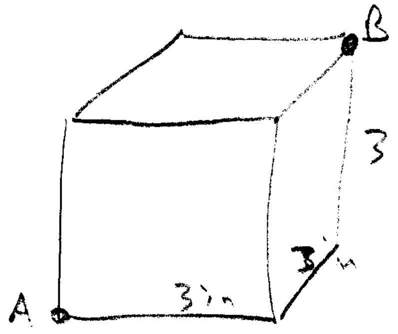
$h^2 + 3.24 = 25$

$h^2 = 21.76$

→ $\boxed{h \sim 4.7m}$

---

(24)



(C)

$V_{cube} = 27 \text{ in}^3$

diagonal distance from A to B

$= \sqrt{3^2 + 3^2 + 3^2}$

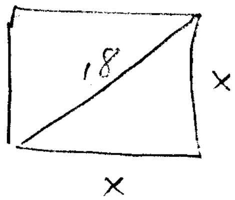
$= \sqrt{9 + 9 + 9} = \sqrt{27}$

$= \boxed{3\sqrt{3}}$

2008 Geometry

(25) diagonal of a square = 18 in. Find Area

(D)



$A = x^2$

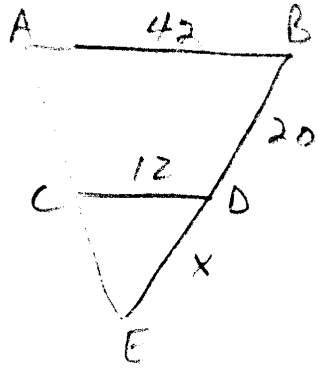
$d^2 = x^2 + x^2$

$18^2 = 2x^2 \rightarrow x^2 = 18^2/2 = \frac{324}{2} = \boxed{162 in^2}$

---

(26)

(A)



AB // CD

Find DE

$\frac{12}{42} = \frac{x}{x+20}$

$12x + 240 = 42x$

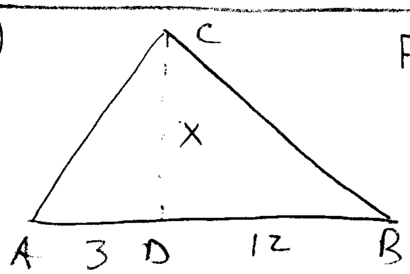
$240 = 30x$

$\boxed{8 = x}$

---

(27)

(C)



Find AC.

$x^2 = (3)(12) \Rightarrow x = 6$

$AC = \sqrt{3^2 + 6^2} = \sqrt{9 + 36} = \sqrt{45}$

$\boxed{AC = 3\sqrt{5}}$

OR $AC = \sqrt{(3)(3+12)} = \sqrt{3(15)} = \sqrt{45}$

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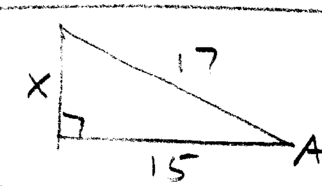
(28) $\cos A = 15/17$

$\sin A = ?$

(B)



$x^2 + 15^2 = 17^2$

$x^2 + 225 = 289$

$x^2 = 64 \Rightarrow x = 8$

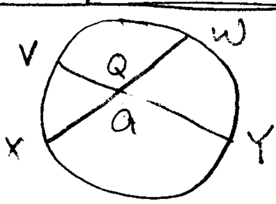
OR 8-15-17 triple

$\boxed{\sin A = 8/17}$

---

(29)

(D)



$m\widehat{XY} = 200°$

$m\widehat{VW} = 66°$

so ..

$a = \frac{1}{2}(m\widehat{VW} + m\widehat{XY})$

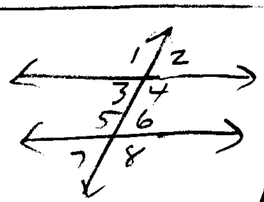
$= \frac{1}{2}(66° + 200°)$

$= \frac{1}{2}(266°) = \boxed{133°}$

---

(30)

(D)



$m\angle 1 = 6x - 5$

$m\angle 7 = 2x + 5$

$m\angle 1 + m\angle 7 = 180°$

$6x - 5 + 2x + 5 = 180$

$8x = 180 \Rightarrow x = 22.5$

$m\angle 1 = 6(22.5) - 5 = \boxed{130°}$

2008 Geometry

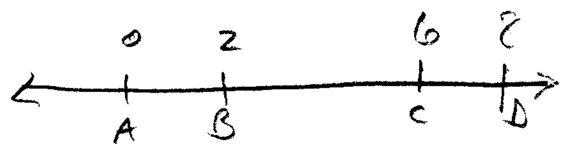
C (31) Supplementary angles at ratio of 3 : 2. Larger angle?

$3x + 2x = 180$

$5x = 180$

$x = 36$ Larger $= 3(36) = \boxed{108^\circ}$

C (32)



3 units from C ⇒ 3 or 9

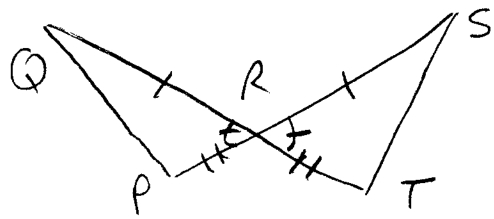
7 units from B ⇒ −5, 9

$\boxed{X = 9}$ → right of D

A (33) Which is false?

$\boxed{\text{An isosceles triangle must have three acute angles}}$

A (34)



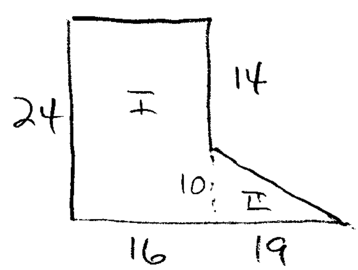
$\triangle QPR \cong \triangle STR$ by $\boxed{SAS}$

C (35) Classify a triangle with sides 3, 5, 6.

$3^2 + 5^2 \overset{?}{} 6^2$

$9 + 25 < 36$ therefore $\boxed{\text{obtuse}}$

B (36)



$\text{Area}_{I} = 24 \times 16 = 384 \text{ ft}^2$

$\text{Area}_{II} = \frac{1}{2}(10)(19) = 95 \text{ ft}^2$

Total area $= 384 + 95 = \boxed{479 \text{ ft}^2}$

D (37) E, F are points. $\boxed{EF = \text{number (distance)}}$

B (38)

A(−5, −2) moved 3 left

B(−3, 2) and 8 up

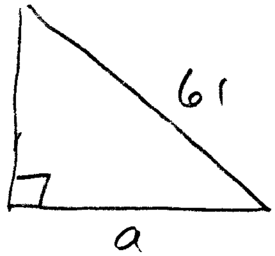
C(−1, −2)

$\boxed{\begin{array}{l} A' = (-8, 6) \\ B' = (-6, 10) \\ C' = (-4, 6) \end{array}}$

2008 Geometry

39

C



Find perimeter (nearest whole number)

$a^2 + 11^2 = 61^2$

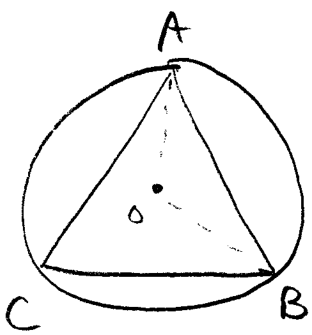
$a^2 + 121 = 3721$

$a^2 = 3600 \rightarrow a = 60$

perimeter $= 11 + 60 + 61 = \boxed{132}$

40

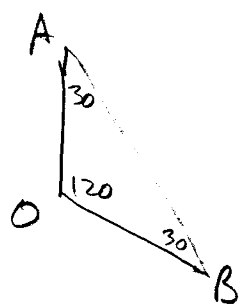
A



Equilateral triangle inscribed inside a circle.

$m\,\widehat{AB} =$ central angle

$= \boxed{120^\circ}$

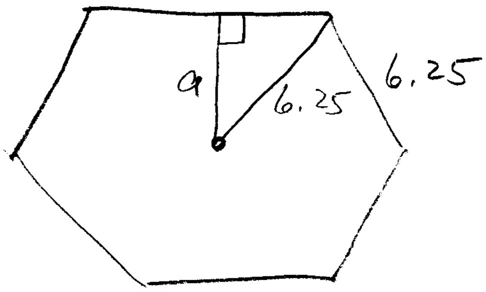


2008 Geometry - TEAM

① Regular hexagon inside a circle of diameter = 12.5 in.

$r = 6.25$

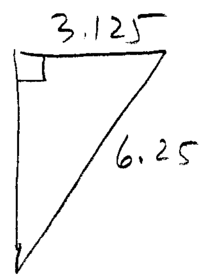
$A_{CIRCLE} = \pi r^2 = \pi (6.25)^2 = 122.72$



$A_{hexagon} = \frac{1}{2} a p$

$= \frac{1}{2}(3.125\sqrt{3})(37.5)$

$= 101.49$



$a = 3.125\sqrt{3}$

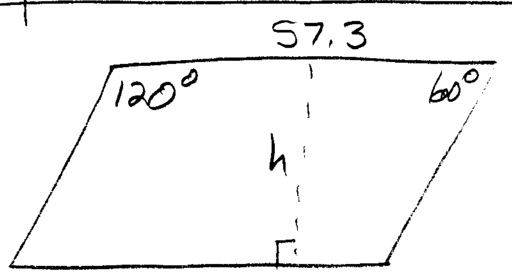
$p = 6(6.25) = 37.5$

Difference = 21.2

to nearest whole number $\boxed{21}$

---

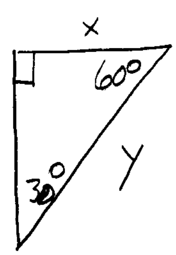
②



Area = 756.36

$A = bh$ so $756.36 = 57.3h$

$h = 13.2$



$x\sqrt{3} = 13.2$

$x = \frac{13.2}{\sqrt{3}}$

$y = 2x = \frac{26.4}{\sqrt{3}} = 15.2$

Perimeter $= 2(57.3) + 2(15.2) = \boxed{145}$

2008 Geometry - TEAM

(3)

Find perimeter.

A $(-2, 3)$

B $(5, -2)$

C $(1, 7)$

$AB = \sqrt{7^2+5^2} = \sqrt{74} = 8.602$

$BC = \sqrt{4^2+9^2} = \sqrt{97} = 9.849$

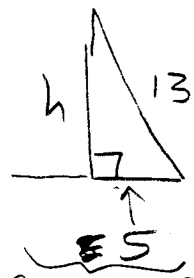
$AC = \sqrt{3^2+4^2} = \sqrt{25} = 5$

23.451

to nearest hundredth : $\boxed{23.45}$

---

(4) Square pyramid: $B = 100$ slant height $= 13$



$h = 12$

$V = \frac{1}{3} B h$

$= \frac{1}{3}(100)(12) = 400$

$SA = B + 2b(\text{slant height})$ $b =$ length of base $= 10$

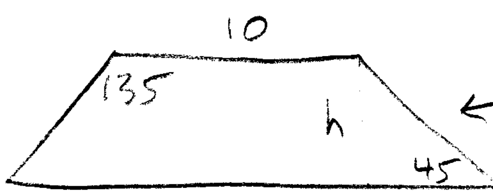
$= 100 + 2(10)(13)$

$= 100 + 260 = 360$

$V + SA = \boxed{760}$

---

(5)



$P = 36 + 16\sqrt{2}$

sides $= 8\sqrt{2}$

bottom $= 26$

$h = 8$

$\text{Area} = \frac{1}{2}(b_1 + b_2) h$

$= \frac{1}{2}(10 + 26)(8) = \boxed{144}$

2008 Geometry – TEAM

(6)

Dodecagon (12 sides): Sum of interior angles $=(n-2)180$

$$\text{Sum} = 10(180) = 1800$$

Ratio of sets of 3 angles is $2:3:4:6$

so multiply each ratio by $(3x)$ and add

$$6x + 9x + 12x + 18x = 1800$$

$$45x = 1800$$

$$x = 40$$

$a = 80$, $b = 120$, $c = 160$ $d = 240$

$$\frac{(a)(c)}{(b)(d)} = \frac{(80)(160)}{(120)(240)} = .\overline{4} = \boxed{\frac{4}{9}}$$

OR $\Rightarrow$ without figuring the exact angles:

$$\frac{(a)(c)}{(b)(d)} = \frac{(6x)(12x)}{(9x)(18x)} = \frac{(\cancel{6}^{2})(\cancel{12}^{2})}{(\cancel{9}_{3})(\cancel{18}_{3})} = \boxed{\frac{4}{9}}$$

---

(7) Cube A: Area of one face $= 25\text{ ft}^2 \Rightarrow$ side $= 5\text{ ft}$

so $\text{Volume}_A = (s)^3 = 125 = x$

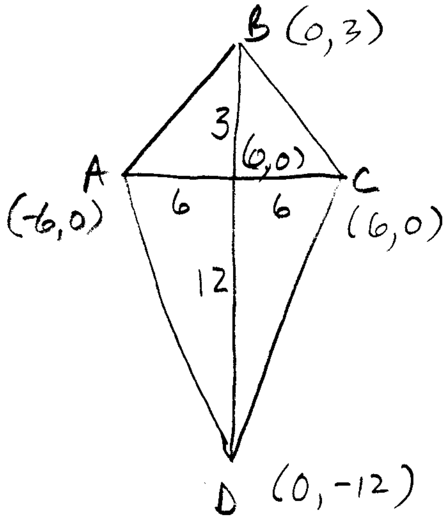
Cube B: $\text{Volume}_B = 343\text{ ft}^3 \Rightarrow$ side $= \sqrt[3]{343} = 7$

total surface area of $B = 6(7^2) = 294 = y$

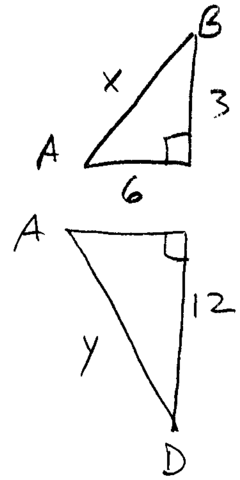
$$xy = (125)(294) = \boxed{36{,}750}$$

2008 Geometry – TEAM

(8)



$$A = \frac{1}{2} d_1 d_2 = \frac{1}{2}(12)(15) = 90$$



$$x = \sqrt{3^2 + 6^2} = \sqrt{45} = 3\sqrt{5}$$

$$y = \sqrt{6^2 + 12^2} = \sqrt{180} = 6\sqrt{5}$$

$$P = 2x + 2y = 2(3\sqrt{5}) + 2(6\sqrt{5}) = 18\sqrt{5}$$

$$\boxed{A + P = 90 + 18\sqrt{5}}$$

---

(9) $A =$ slope of $8x - 3y = -5 \Rightarrow y = 8/3\, x + 5/3$

$A = 8/3$

$B =$ slope of parallel to $5x + 4y = 100 \Rightarrow y = -5/4\, x + 25$

$B = -5/4$

$C =$ slope of perpendicular to $5x - 2y = -1.5 \Rightarrow y = -\frac{5}{2}x + \frac{3}{4}$

$C = 2/5$

$$A - B + C = \frac{8}{3} - \left(\frac{-5}{4}\right) + \frac{2}{5} = \frac{160 + 75 + 24}{60} = \frac{259}{60}$$

as mixed number $\boxed{4\frac{19}{60}}$

2008 Geometry – TEAM

(12)

$G(-4,4)$, $H(4,0)$ } eqn for $\overleftrightarrow{GH}$: $\quad m = \frac{4-0}{-4-4} = \frac{4}{-8} = -\frac{1}{2}$

$y = -\frac{1}{2}x + b$ substitute $(4,0)$

$0 = -\frac{1}{2}(4) + b$

$b = 2 \implies y = -\frac{1}{2}x + 2$

$A = -1/2$, $B = 2$

Eqn of $\perp$ bisector:

slope $= 2$ (opposite reciprocal)

midpoint $= \left(\frac{-4+4}{2}, \frac{4+0}{2}\right) = (0,2)$

eqn $\implies 2 = 2(0) + b \implies b = 2$

$y = 2x + 2$

$C = 2$, $D = 2$

$\frac{AB}{CD} = \frac{(-1/2)(2)}{(2)(2)} = \boxed{-1/4}$

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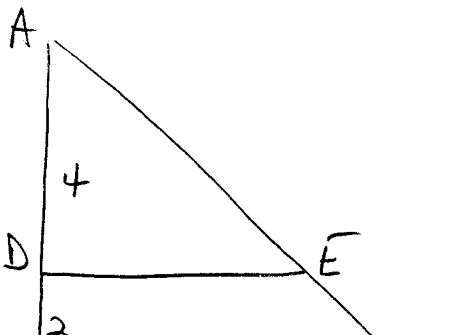
(13)

Invisible check



$\frac{AD}{DE} = \frac{AB}{BC} \implies \frac{4}{DE} = \frac{6}{8}$

$6DE = 32$

$DE = \frac{32}{6} = \frac{16}{3}$

$AC^2 = 6^2 + 8^2$

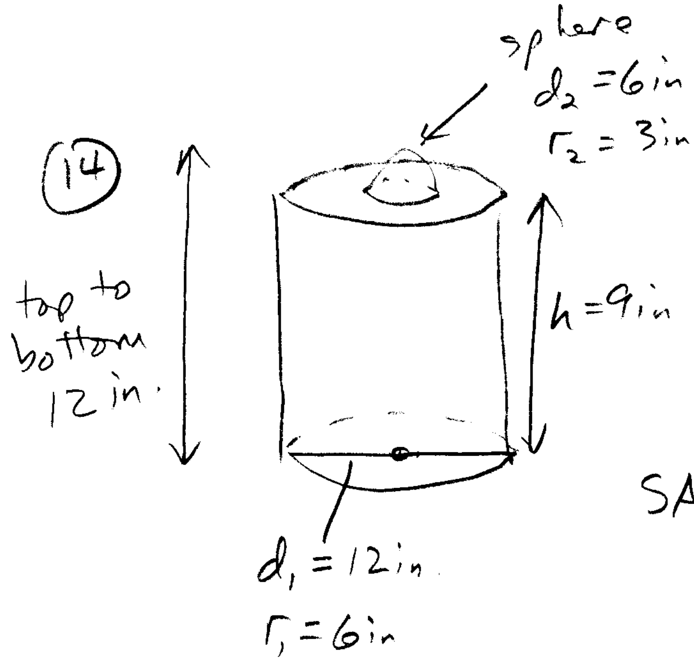
$AC = 10$

$\frac{AD}{AE} = \frac{AB}{AC} \implies \frac{4}{AE} = \frac{6}{10} \implies 6AE = 40 \quad AE = \frac{40}{6} = \frac{20}{3}$

$EC = AC - AE = 10 - \frac{20}{3} = \frac{30}{3} - \frac{20}{3} = \frac{10}{3}$

$AE + DE - EC = \frac{20}{3} + \frac{16}{3} - \frac{10}{3} = \frac{26}{3} = \boxed{8\,2/3}$

(14)



Find $V_{TOTAL} + SA_{TOTAL}$

$$V_{cylinder} = \pi r_1^2 h = \pi(6)^2(9) = 324\pi$$

$$V_{hemisphere} = \tfrac{1}{2}\left(\tfrac{4}{3}\pi r_2^3\right) = \tfrac{1}{2}\left(\tfrac{4}{3}\right)(\pi)(3)^3 = 18\pi$$

$$SA_{cyl} = \underbrace{\pi r_1^2}_{\text{bottom}} + \underbrace{2\pi r_1 h}_{\text{side}} + \underbrace{\left[\pi r_1^2 - \pi r_2^2\right]}_{\text{top}}$$

$$= \pi(6)^2 + 2\pi(6)(9) + \left[\pi(6)^2 - \pi(3)^2\right]$$

$$= 36\pi + 108\pi + 36\pi - 9\pi$$

$$= 171\pi$$

$$SA_{hemisphere} = \tfrac{1}{2}(4\pi r_2^2) = \tfrac{1}{2}(4)(\pi)(3)^2 = 18\pi$$

$$V_{TOT} + SA_{TOT} = 324\pi + 18\pi + 171\pi + 18\pi = \boxed{531\pi}$$

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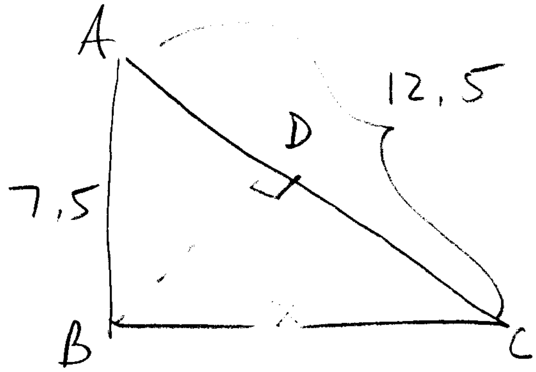
(15)



$$AB^2 = (AD)(AC)$$
$$7.5^2 = (AD)(12.5)$$
$$AD = 4.5$$
Geometric Mean

$$BC^2 + 7.5^2 = 12.5^2$$
$$BC^2 + 56.25 = 156.25$$
$$BC^2 = 100 \; ; \quad BC = 10$$

$$BC^2 = (CD)(AD)$$
$$10^2 = (CD)(12.5)$$
$$CD = 8$$
Geometric Mean

$$BD^2 = (AD)(DC)$$
$$BD^2 = (4.5)(8)$$
$$BD^2 = 36$$
$$BD = 6$$

$$(AD)(DC)(BD)(BC)$$
$$(4.5)(8)(6)(10) = \boxed{2160}$$