ALGEBRA - 2008

D ① $-\dfrac{14x}{15} - \dfrac{x}{3} = \dfrac{-14x}{15} - \dfrac{5x}{15} = \boxed{\dfrac{-19x}{15}}$

C ② \$89.95 on sale for 15% off with 6% tax added

$(89.95)(.85)(1.06) = \boxed{\$81.07}$

OR estimation $(90)(.85)(1.06) \cong \$81.09$

A ③ Solve for x:

$\dfrac{x}{a} + \dfrac{y}{b} = 1 \Rightarrow \dfrac{x}{a} = 1 - \dfrac{y}{b} \Rightarrow x = a\left(1 - \dfrac{y}{b}\right)$

$= \boxed{a - \dfrac{ay}{b}}$

C ④ Peanuts \$6.75/lb
Cashews \$9.50/lb
32 more pounds of peanuts
Sales = \$1012.25
How many peanuts?

Let x = peanuts, y = cashews

$6.75x + 9.50y = 1012.25$

$x = y + 32$

$6.75x + 9.50(x - 32) = 1012.25$

$6.75x + 9.50x - 304 = 1012.25$

$16.25x = 1316.25$

$x = 1316.25/16.25 = \boxed{81}$

B ⑤ Find 7th term:

$1\tfrac{4}{9},\ 2\tfrac{1}{9},\ 2\tfrac{7}{9},\ 3\tfrac{4}{9},\ \ldots$

change to improper fractions

$\dfrac{13}{9}, \dfrac{19}{9}, \dfrac{25}{9}, \dfrac{31}{9}, \ldots \dfrac{37}{9}, \dfrac{43}{9}, \boxed{\dfrac{49}{9} = 5\tfrac{4}{9}}$

add $\dfrac{6}{9}$

ALGEBRA – 2008

(6) Which is equivalent to $4x + 7y = -66$

solve for y: $y = -\frac{4}{7}x - \frac{66}{7}$

(D) answers are in point-slope form, so slope needs to be $-4/7$ (correct for A, B, and D).

Check points: (A) $(-6, 6)$ (B) $(6, -6)$ (C) $(-6, -6)$

(A)? $-\frac{4(-6)}{7} - \frac{66}{7} = \frac{24}{7} - \frac{66}{7} = -\frac{42}{7} \neq +6$ (not A)

(B)? $\frac{(-4)(6)}{7} - \frac{66}{7} = \frac{-24}{7} - \frac{66}{7} = \frac{-90}{7}$ (not B)

(C)? $\frac{(-4)(-6)}{7} - \frac{66}{7} = \frac{24}{7} - \frac{66}{7} = \boxed{\frac{-42}{7} = -6}$ correct

---

(7) (A)

$0.3(2j+2) > 2.4 - (-0.4j - 3)$

$.6j + .6 > 2.4 + .4j + 3$

$.6j + .6 > 5.4 + .4j$

$-.4j - .6 \quad -.6 \quad -.4j$

$\frac{.2j}{.2} > \frac{4.8}{.2} \Rightarrow \boxed{j > 24}$

---

(8) (C) Two angles are complementary (sum = 90)

One is 32° more than other. What are 2 angles

$x + (x + 32) = 90°$

$2x + 32 = 90°$

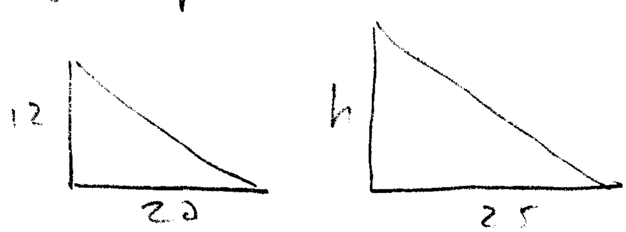
$2x = 58$

$\boxed{x = 29° \text{ and } 29 + 32 = 61°}$

---

(9) (D) A 12-foot pole casts 20-foot shadow.

Another pole casts 25-foot shadow



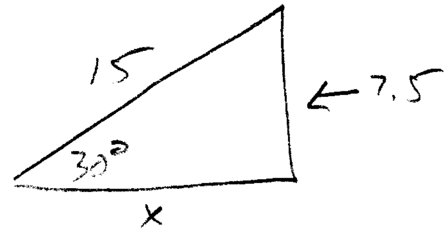
$\frac{12}{20} = \frac{h}{25}$

Cross-multiply: $300 = 20h$

$h = 300/20 = \boxed{15 \text{ ft}}$

ALGEBRA

(C) (10) Find x:



$x = 7.5\sqrt{3}$

$\approx (7.5)(1.7) \approx 12.75$

$\boxed{x = 13}$

(D) (11) $\sqrt{0.144} \rightarrow$ between $\left.\begin{array}{l} \sqrt{.09} = .3 \\ \sqrt{.16} = .4 \end{array}\right\} \sim \boxed{0.38}$

(B) (12) Find median:

$2.1, 2.1, \underbrace{2.1, 4.8}, 4.8, 5.7$

$\text{Median} = \dfrac{2.1 + 4.8}{2} = \dfrac{6.9}{2} = \boxed{3.45}$

(A) (13) Solve for x: $\left(\dfrac{1}{8} - \dfrac{3}{4}x = \dfrac{1}{16}\right) 16 \Rightarrow 2 - 12x = 1$

mult. by

$\begin{array}{r} 2 - 12x = 1 \\ -2 \qquad\quad -2 \\ \hline -12x = -1 \end{array} \Rightarrow \boxed{x = 1/12}$

---

(14) A (-6, 4) B (-2, -2) Find equation for perpendicular bisector of $\overline{AB}$

(D)

slope of $\overline{AB} = \dfrac{-2-4}{-2 - -6} = \dfrac{-6}{4} = \dfrac{-3}{2}$

slope of $\perp$ bisector $= 2/3$

midpoint of $\overline{AB} = \left(\dfrac{-6 + -2}{2}, \dfrac{4 + -2}{2}\right) = \left(\dfrac{-8}{2}, \dfrac{2}{2}\right) = (-4, 1)$

plug into $y = mx + b \Rightarrow 1 = \frac{2}{3}(-4) + b$

$b = 1 + \frac{8}{3} = \frac{11}{3} \Rightarrow \boxed{y = \frac{2}{3}x + \frac{11}{3}}$

---

(15) exact distance between (9, -2) and (12, -14)

(A)

$d = \sqrt{(9-12)^2 + (-2 - -14)^2} = \sqrt{(-3)^2 + (12)^2} = \sqrt{9 + 144} = \sqrt{153}$

$= \sqrt{9 \cdot 17} = \boxed{3\sqrt{17}}$

ALGEBRA - 2008

(16) (B)

$$\frac{2}{2-x} = \frac{-4}{x-5}$$

Cross-multiply: $2x - 10 = -8 + 4x$

$-2x + 8 \quad +8 - 2x$

$-2 = 2x$

$\boxed{x = -1}$

---

(17) (C)

$|3x + 11| > 1$

$3x + 11 > 1$ or $3x + 11 < -1$

$3x > -10$ $\qquad 3x < -12$

$x > -\frac{10}{3} \Rightarrow \boxed{x > -3.\overline{3}}$ $\qquad \boxed{x < -4}$

---

(18) (D)

$3x + 2y = 8$

$\left(y = -\frac{3}{2}x + 5\right)2 \Rightarrow 2y = -3x + 10$, $3x + 2y = 10$

$+3x \quad +3x$

multiply by

these are parallel lines, so $\boxed{\text{No solution}}$

---

(19) (A)

$b + c = 4 \longrightarrow c = 4 - b$

$2a + 4b - c = -3$

$3a - b = 10 \longrightarrow a = \frac{b+10}{3}$

substitute into middle eqn.

$2\left(\frac{b+10}{3}\right) + 4b - (4 - b) = -3$

$\frac{2b+20}{3} + 4b - 4 + b = -3$

mult. by 3: $2b + 20 + 12b - 12 + 3b = -9$

$17b + 8 = -9$

$17b = -17$

$\boxed{b = -1} \rightarrow$ (A) or (C)

substitute into 1st eqn: $-1 + c = 4 \Rightarrow c = 5$ so (A) or (C)

substitute into 3rd eqn: $3a + 1 = 10 \Rightarrow 3a = 9$, $a = 3$

$\boxed{(3, -1, 5)}$

---

(20) (A)

$$\left(\frac{3y^{-2}}{-2x}\right)^{-3} = \left(\frac{-2x}{3y^{-2}}\right)^{3} = \frac{(-2)^3 x^3}{(3^3)\, y^{-6}} = \frac{-8x^3}{27y^{-6}}$$

$$= \boxed{\frac{-8x^3y^6}{27}}$$

ALGEBRA 2008

D (21) $\sqrt{98} - \sqrt{72} + \sqrt{32}$

$\sqrt{49\cdot 2} - \sqrt{36\cdot 2} + \sqrt{16\cdot 2} = 7\sqrt{2} - 6\sqrt{2} + 4\sqrt{2} = \boxed{5\sqrt{2}}$

C (22) $\dfrac{(2-\sqrt{5})(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})} = \dfrac{4-4\sqrt{5}+5}{4-5} = \dfrac{9-4\sqrt{5}}{-1} = \boxed{-9+4\sqrt{5}}$

D (23) $\frac{1}{32} = 4^{x+1}$

$2^{-5} = 2^{2(x+1)} \Rightarrow$ $-5 = 2x+2$, $\;-2 \quad -2$, $-7 = 2x \Rightarrow \boxed{x = -7/2}$

C (24) $3^{-2}(4^3 \div 8 - 4^0 \times (-7))$

$\frac{1}{9}(64 \div 8 - 1 \times (-7)) \Rightarrow \frac{1}{9}(8 - {}^-7) = \frac{15}{9} = \boxed{\frac{5}{3}}$

---

(25) Determinant of $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix} \begin{matrix} 1 & 2 \\ -1 & 2 \\ 2 & 1 \end{matrix}$

C

$\det = (1)(2)(1) + (2)(-1)(2) + (3)(-1)(1) - (2)(2)(3)$
$\quad - (1)(-1)(1) - (1)(-1)(2)$

$= 2 - 4 - 3 - 12 + 1 + 2 = \boxed{-14}$

---

(26) $12y^2 - 13y - 25 = 10$

$\qquad\qquad -10 \quad -10$

B

$12y - 13y - 35 = 0$

$(4y+5)(3y-7) = 0$

$4y+5=0 \Rightarrow$ $y = -5/4$

$3y-7=0 \Rightarrow$ $y = 7/3$

ALGEBRA 2008

D (28) discriminant of $y = x^2 - 4x + 1$

$\hookrightarrow b^2 - 4ac = (-4)^2 - 4(1)(1) = 16 - 4 = \boxed{12}$

B (29) $-9x^4 + 66x^3 - 72x^2$

$-3x^2(3x^2 - 22x + 24)$

$\boxed{-3x^2(3x - 4)(x - 6)}$

A (30) $x^2 - \frac{4}{25} \Rightarrow \boxed{(x + \frac{2}{5})(x - \frac{2}{5})}$

difference of squares

A (31) What is 0.4% of 800?

$(.004)(800) = \boxed{3.2}$

C (32) What % of 20 is 31?

$\frac{x}{100} = \frac{31}{20} \Rightarrow x = \boxed{155\%}$

B (33) 200% of what number is 5?

$\frac{200}{100} = \frac{5}{x} \Rightarrow x = \boxed{2.5}$

B (34) Eqn for direct variation through $(-2, 9)$. $\}$ $y = mx$

Means it goes through $(0,0)$

$m = -9/2 \Longrightarrow \boxed{y = -9/2\, x}$

D (35) slope $= -1/5$ through $(6, -4)$ $\}$ in point slope form: $y - y_1 = m(x - x_1)$

$\boxed{y + 4 = -\frac{1}{5}(x - 6)}$

C (36) vertex of $y = 3x^2 + 6x$

vertex at $x = -\frac{b}{2a} = -\frac{6}{2(3)} = -\frac{6}{6} = -1$

plug into eqn: $y = 3(-1)^2 + 6(-1)$

$= 3 - 6 = -3$

$\boxed{(-1, -3)}$

ALGEBRA - 2008

(A) (37) axis of symmetry for $y = -5x^2 + x + 4 \Rightarrow$ eqn is $x =$ vertex

$$\text{vertex} = -\frac{b}{2a} = -\frac{1}{2(-5)} = -\frac{1}{-10} = 1/10 \qquad \boxed{x = 1/10}$$

(B) (38) Factor $2p^3 - 4p^2 + 2p - 4$

$$2p^2(p-2) + 2(p-2) = (2p^2+2)(p-2) = \boxed{2(p^2+1)(p-2)}$$

(C) (39) $$\frac{3y}{4y-8} \div \frac{9y}{2y^2-4y} = \frac{3y}{4(y-2)} \div \frac{9y}{2y(y-2)}$$

$$\frac{\cancel{3y}}{\underset{2}{\cancel{4}}\cancel{(y-2)}} \times \frac{\cancel{2}y\cancel{(y-2)}}{\underset{3}{\cancel{9y}}} = \boxed{\frac{y}{6}}$$

(B) (40) Solve ~~for~~ r:

$$\frac{h}{p-m} = \frac{\frac{r}{t}(p-m)}{p-m} \Rightarrow t \cdot \frac{h}{p-m} = \frac{r}{t} \cdot t$$

$$\boxed{\frac{ht}{p-m} = r}$$

2008 ALGEBRA TEAM

1.

$$9.6 \text{ Euro} \times \frac{\$1}{0.64 \text{ Euro}} = 15$$

$$9.6 \text{ Euro} \times \frac{7.09 \text{ Yuan}}{0.64 \text{ Euro}} = 106.35$$

$$9.6 \text{ Euro} \times \frac{31.35 \text{ Baht}}{0.64 \text{ Euro}} = 470.25$$

$$\begin{array}{r} 9.60 \\ +\ 15.00 \\ +106.35 \\ +\ 470.25 \\ \hline \boxed{601.20} \end{array}$$

2.

$f(x) = -13.9x$

$g(x) = -8x - 3$

$$g(f(3)) = g(-13.9(3)) = g(-41.7)$$
$$= -8(-41.7) - 3 = 330.6$$
$$f(g(-\tfrac{1}{2})) = f[-8(-\tfrac{1}{2}) - 3] = f[1]$$
$$= -13.9(1) = -13.9$$
$$g(f(3)) - f(g(-1/2)) = 330.6 - (-13.9) = \boxed{344.5}$$

---

3. $C = 40{,}000$ $\quad r = \frac{C}{2\pi} = \frac{40000}{2\pi} \approx 6400$ (nearest hundred)

$$R_{\text{to orbit}} = r + 36000 = 42400 \text{ km}$$
$$C = 2\pi R = 2\pi(42400) = 266407 \text{ km}$$
$$24 \text{ hrs} \times \frac{3600 \text{ sec}}{1 \text{ hr}} = 86400 \text{ sec}$$

$$\text{speed} = \frac{266407}{86400} = 3.08 \sim \boxed{3.1 \text{ km/sec}}$$

nearest tenth

2008 ALGEBRA TEAM

(4) $G = 2J - 20$

$S = \frac{1}{2}G + 2 \Rightarrow S = \frac{1}{2}(2J-20)+2 = J-10+2 = J-8$

$S + G + J = 112$

substitution: $(J-8) + (2J-20) + J = 112$

$4J - 28 = 112$

$4J = 140 \Rightarrow J = 35$

$G = 2(35) - 20 = 50$

$S = \frac{1}{2}(50) + 2 = 27$

positive difference of oldest − youngest $= 50 - 27 = \boxed{23}$

---

(5)

5th 6th

$0.5, \frac{1}{3}, \frac{1}{6}, 0, \ldots$ (subtract $\frac{1}{6}$) $\Rightarrow$ $-\frac{1}{6}$ $-\frac{2}{6} = -\frac{1}{3}$

$17.6, -8.8, 4.4, -2.2, \ldots$ (mult by $-\frac{1}{2}$) $\Rightarrow$ 1.1 -0.55

$A(n) = -2 + (n-2)(-1.6) \Rightarrow A(6) = -2 + (4)(-1.6) = -8.4$

$B(n) = \frac{1}{9}(-3)^{n-1} \Rightarrow B(6) = \frac{1}{9}(-3)^5 = -27$

$(-\frac{1}{3})(-0.55)(-8.4)(-27) = \boxed{41.58}$

2008 ALGEBRA – TEAM

(6) $A = \text{slope } (3,-7)\ (-4,3)$ $\quad m = \dfrac{3 - -7}{-4-3} = \dfrac{10}{-7} = -\dfrac{10}{7} = A$

$B = \text{slope of parallel to } 3x - 5y = 10$ $\quad B = \dfrac{3}{5}$

$\underline{-3x \qquad -3x}$

$-5y = -3x + 10 \Rightarrow y = \dfrac{3}{4}x - \dfrac{5}{2}$

$C = \text{slope of perpendicular to } 5x + 2y = 25$ $\quad C = 2/5$

$\underline{-5x \qquad -5x}$

$2y = -5x + 25$ (opposite, reciprocal)

$y = -5/2\,x + 25/2$

$A + B + C = \left(-\dfrac{10}{7} + \dfrac{3}{5} + \dfrac{2}{5}\right) = -\dfrac{50}{35} + \dfrac{21}{35} + \dfrac{14}{35} = -\dfrac{15}{35}$

$= \cancel{-\dfrac{200}{140} + \dfrac{105}{140} + \dfrac{56}{140} = \boxed{-\dfrac{39}{140}}} \qquad = \boxed{-\dfrac{3}{7}}$

---

(7) 12 coins (nickels and quarters) total \$1.20

$n + q = 12 \Rightarrow n = 12 - q$

$.05n + .25q = 1.20 \Rightarrow$ mult by 20: $n + 5q = 24$

substitution: $12 - q + 5q = 24$ $\quad q = 3$, $n = 9$

$\underline{-12 \qquad\qquad -12}$

$4q = 12$

A boat goes 12 miles downstream in 2 hours and upstream in 3 hours

$\begin{cases} 12 = (B+C)2 & \text{- downstream} \\ 12 = (B-C)3 & \text{- upstream} \end{cases}$

$6 = B + C$

$\underline{4 = B - C}$

$10 = 2B$

$B = 5$, so $C = 1$

$\dfrac{BC}{NQ} = \dfrac{(5)(1)}{(9)(3)} = \boxed{\dfrac{5}{27}}$

2008 ALGEBRA - TEAM

(8) $x = -2$

$$A = \left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^{2} = \frac{25}{9}$$

$$B = \frac{1}{2}(-3)^{-2} = \frac{1}{2}\left(-\frac{1}{3}\right)^{2} = \frac{1}{18}$$

$$C = \left(-\frac{1}{3}\right)^{-2} = \left(\frac{-3}{1}\right)^{2} = 9$$

$$D = -\frac{1}{9}(3)^{-2} = -\frac{1}{9}\left(\frac{1}{3}\right)^{2} = -\frac{1}{81}$$

$$A - B \div C \div D$$

$$\frac{25}{9} - \underbrace{\frac{1}{18} \div 9 \div \left(-\frac{1}{81}\right)}$$

$$\frac{25}{9} - \underbrace{\left(\frac{1}{\cancel{18}_{2}} \cdot \frac{1}{\cancel{9}_{1}} \cdot \frac{-\cancel{81}^{1}}{1}\right)}$$

$$\frac{25}{9} - \left(-\frac{1}{2}\right) = \frac{25}{9} + \frac{1}{2}$$

$$= \frac{50}{18} + \frac{36}{18} = \frac{86}{18} = \boxed{\frac{43}{9}}$$

---

(9) Q, R = solutions to $10x^2 - x = 3 \Rightarrow 10x^2 - x - 3 = 0$

$\qquad\qquad -3 \quad -3$

$$(5x - 3)(2x + 1) = 0$$

$$Q = 3/5 = 0.6$$

(S, T) = vertex of $10x^2 - x - 3 = 0$ $\qquad R = -1/2 = -0.5$

$$S = -\frac{b}{2a} = -\frac{-1}{2(10)} = \frac{1}{20} = 0.05$$

substitute: $10\left(\frac{1}{20}\right)^2 - \frac{1}{20} - 3 = \frac{10}{400} - \frac{1}{20} - 3$

$$= \frac{1}{40} - \frac{2}{40} - \frac{120}{40} = \frac{-121}{40} = 3.025$$

$$(Q+R)ST = (0.6 - 0.5)(-3)(3.025)$$

$$= (0.1)(-3)(3.025)$$

$$= \boxed{-0.9075}$$

2008 ALGEBRA - TEAM

(10)

$(8.3, -9.4) \quad (-3.7, 6.6)$

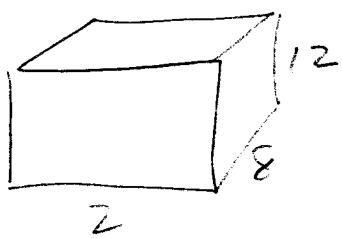
$$L = \text{distance} = \sqrt{(-3.7-8.3)^2 + (6.6 - {}^-9.4)^2}$$
$$= \sqrt{(-12)^2 + (16)^2} = \sqrt{400} = 20$$

$$(M, N) = \text{midpoint} = \left(\frac{8.3 + {}^-3.7}{2}, \frac{-9.4 + 6.6}{2}\right) = (2.3, -1.4)$$

$$L + M + N = 20 + 2.3 + {}^-1.4 = \boxed{20.9}$$

---

(11)

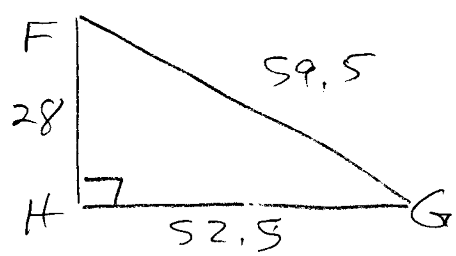


$$d_1 = \sqrt{2^2 + 8^2} = \sqrt{4+64} = \sqrt{68}$$
$$d_1 = 2\sqrt{17}$$
$$d_2 = \sqrt{2^2 + 12^2} = \sqrt{4 + 144} = \sqrt{148}$$
$$d_2 = 2\sqrt{37}$$
$$d_3 = \sqrt{8^2 + 12^2} = \sqrt{64 + 144} = \sqrt{208}$$
$$d_3 = 4\sqrt{13}$$

sum of six diagonals $= \boxed{4\sqrt{17} + 4\sqrt{37} + 8\sqrt{13}}$

---

(12)



$$\sin F = \frac{52.5}{59.5} = \frac{15}{17}$$

$$\cos F = \frac{28}{59.5} = \frac{8}{17}$$

$$\tan F = \frac{52.5}{28} = \frac{15}{8}$$

$$\frac{\sin F + \cos F}{\tan F} = \frac{\frac{15}{17} + \frac{8}{17}}{\frac{15}{8}} = \frac{23}{17} \div \frac{15}{8} = \frac{23}{17} \cdot \frac{8}{15} = \boxed{\frac{184}{255}}$$

2008 ALGEBRA TEAM

(13)

$F = 16\tfrac{2}{3}\%$ of $240 = \frac{1}{6}(240) = 40$

$8 = G\%$ of $125 \Rightarrow 8 = \frac{G}{100} \cdot 125 \Rightarrow G = \frac{800}{125} = 6.4$

$19 = 95\%$ of $H \Rightarrow 19 = .95H \Rightarrow H = \frac{19}{.95} = 20$

$$\frac{FG}{H} = \frac{(40)(64)}{20} = \boxed{128}$$

---

(14) A, B are solutions to $|4k-2| = 11$

$$\begin{array}{ll} 4k-2 = 11 & 4k-2 = -11 \\ \quad +2 \;\; +2 & \quad +2 \;\; +2 \\ \hline 4k = 13 & 4k = -9 \\ A = \frac{13}{4} & B = -\frac{9}{4} \end{array}$$

C, D are solutions to $|2c+1| - 4 = 13$

$$\begin{array}{l} |2c+1| - 4 = 13 \\ \qquad\quad +4 \;\; +4 \\ \hline |2c+1| = 17 \end{array}$$

$$\begin{array}{ll} 2c+1 = 17 & 2c+1 = -17 \\ \quad -1 \;\; -1 & \quad -1 \;\; -1 \\ \hline 2c = 16 & 2c = -18 \\ C = 8 & D = -9 \end{array}$$

$$A+B+C+D = \frac{13}{4} + \frac{-9}{4} + 8 + -9$$
$$= \frac{4}{4} + 8 + -9 = 1 + 8 + -9 = \boxed{0}$$

2008 ALGEBRA - TEAM

15

3 red, 4 blue, 5 white, 2 black, 1 orange => 15 total

$P = P(\text{not white}) = \frac{10}{15} = \frac{2}{3}$

$Q = P(\text{red or blue}) = \frac{7}{15}$

$R = P(\text{two marbles: 1 black, 1 orange})$

$= \frac{\cancel{2}^{1}}{15} \cdot \frac{1}{\cancel{14}_{7}} = \frac{1}{105}$

$$\frac{P+Q}{R} = \frac{\frac{2}{3} + \frac{7}{15}}{\frac{1}{105}} = \frac{\frac{10}{15} + \frac{7}{15}}{\frac{1}{105}} = \frac{17}{15} \cdot \frac{105}{1} = \boxed{119}$$